



Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
Third Semester B.Tech Degree (S,FE) Examination December 2020

Course Code: CS201

Course Name: DISCRETE COMPUTATIONAL STRUCTURES

Max. Marks: 100

Duration: 3 Hours

PART A*Answer all questions, each carries 3 marks.*

Marks

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| 1 | Draw the Hasse diagram of posets under the partial order relation of divisibility
$A = \{1, 2, 3, 5, 6, 10, 15, 30\}$ | (3) |
| 2 | Determine whether the relation $R = \{(a, b) a \geq b\}$ on the set of real numbers is an equivalence relation. | (3) |
| 3 | In how many ways can letters in the English alphabet be arranged so that there are exactly 7 letters between the letters 'a' and 'b'. | (3) |
| 4 | Among the first 500 positive integers, determine the integers which are not divisible by 2, nor by 3, nor by 5. | (3) |

PART B*Answer any two full questions, each carries 9 marks.*

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| 5 | a) Let $f(x) = x + 2$, $g(x) = x - 2$ and $h(x) = 3x$ for $x \in \mathbb{R}$, where \mathbb{R} is the set of real numbers. Find gof , fog , fof , gog , foh , hog , hoh and fohog | (4) |
| | b) If the function f is defined by $f(x) = x^2 + 1$ on the set $\{-2, -1, 0, 1, 2\}$, find the range of f . | (5) |
| 6 | a) Show that the set \mathbb{N} of natural numbers is a semigroup under the operation $x * y = \max(x, y)$. Is it a monoid? | (4) |
| | b) 8 scientists and 5 politicians take part in a conference. In how many ways can they be seated in a single row if (i) no 2 politician must sit together (ii) no 2 scientist must sit together. | (5) |
| 7 | a) Solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$, $n \geq 2$ and $a_0 = 1$ and $a_1 = 4$ | (5) |
| | b) Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. | (4) |

PART C*Answer all questions, each carries 3 marks.*

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| 8 | Define group homomorphism. | (3) |
| 9 | How many proper subgroups will be there for a group of order 11? Justify your Answer. | (3) |
| 10 | Let (L, \leq) be a lattice and a, b, c, d elements of L . Prove that if $a \leq c$ and $b \leq d$ then $a \vee b \leq c \vee d$ | (3) |
| 11 | Define a complemented lattice. Give an example. | (3) |

PART D

Answer any two full questions, each carries 9 marks.

- 12 a) What is a complete lattice? Give an example. (4)
 b) Show that the set of all positive rational numbers Q^+ forms an abelian group under the operation $*$ defined by $a*b=(ab)/2$ for $a,b \in Q^+$. (5)
- 13 a) Define a Boolean algebra. Illustrate a two element Boolean Algebra with an example. (4)
 b) Let $H = \{0, 3, 6\}$ in Z_9 under addition. What are the cosets of H in Z_9 ? (5)
- 14 a) Verify that the set $\{0, 1, 2, 3, 4, 5\}$ under addition and multiplication modulo 6 is group or not. (4)
 b) $A = \{2, 3, 4, 6, 12, 18, 24, 36\}$ with partial order of divisibility. Determine whether the POSET is a lattice or not. (5)

PART E

Answer any four full questions, each carries 10 marks.

- 15 a) Without using truth tables prove that $\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$ (5)
 b) Suppose x is a real number. Consider the statement "If $x^2 = 4$, then $x = 2$." Construct the converse, inverse, and contrapositive. (5)
- 16 a) Prove that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent. (5)
 b) Prove that $(\exists x) (P(x) \wedge Q(x)) \Rightarrow (\exists x) P(x) \wedge (\exists x) Q(x)$. (5)
- 17 a) Show that the premises "A student in this class has not read the book" and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book". (5)
 b) Show that the premises, "It is not sunny this afternoon and it is colder than yesterday", "We will go swimming only if it is sunny", "If we do not go swimming, then we will take a canoe trip", and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset". (5)
- 18 a) Show that $(x) (P(x) \rightarrow Q(x)) \wedge (x) (Q(x) \rightarrow R(x)) \Rightarrow (x) (P(x) \rightarrow R(x))$ (5)
 b) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\neg M$. (5)
- 19 a) Show that $(\exists x) M(x)$ follows logically from the premises $(x) (H(x) \rightarrow M(x))$ and $(\exists x) H(x)$ (5)
 b) Show that $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$ is a tautology. (5)
- 20 a) Prove by contradiction "If $3n + 2$ is an odd integer, then n is odd". (5)
 b) Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$ (5)

